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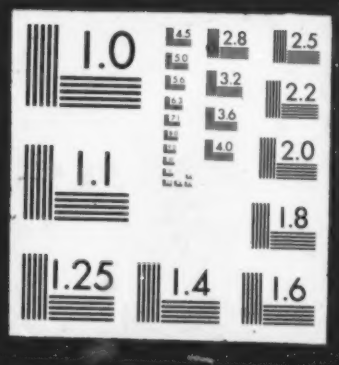

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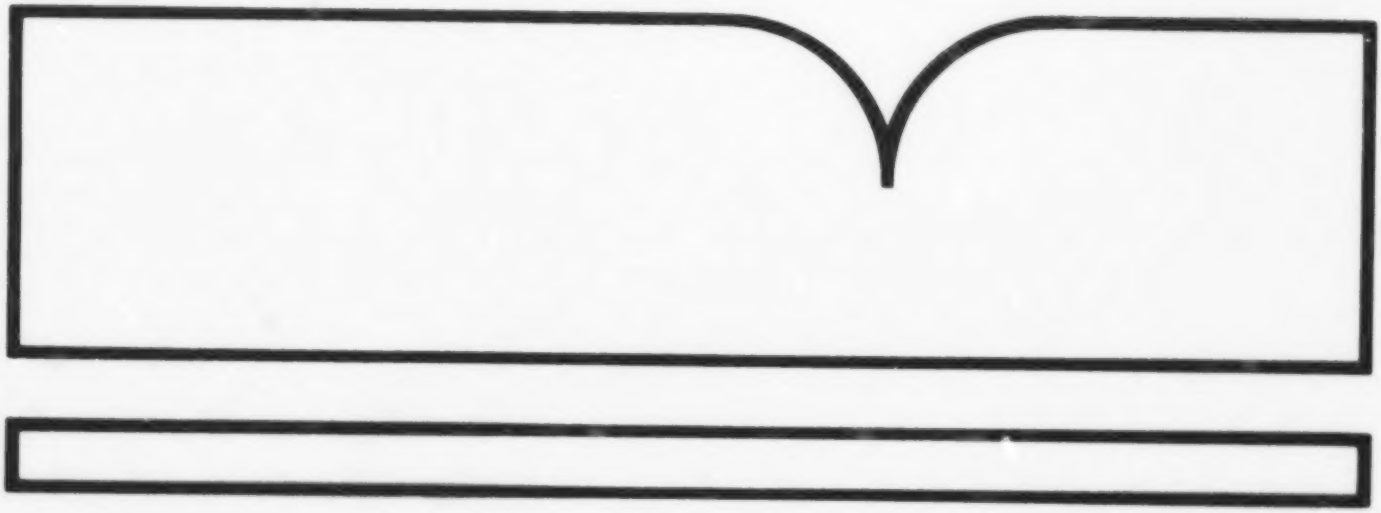

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MULTI-SCALING AND EXPONENTIAL FITTING IN AUTOCORRELATION ANALYSIS

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16. ABSTRACT

Autocorrelation functions contain information about turbulence, including eddy sizes and strengths. In order to make that information available to Gaussian diffusion models, the autocorrelation function must itself be modeled by reasonably simple mathematical terms. This paper postulates a way to describe turbulence which leads to a method for fitting the autocorrelation function by a series of exponential terms. It shows that this procedure leads to simple, mathematically and physical consistent definitions for the transformed autocorrelation and the spectra. An example is given to show the utility of the method.

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1. Introduction

Correlation functions contain a great deal of information about turbulence, including eddy sizes and the strength of the turbulence as measured by the variance. Usually we obtain this information from the Fourier transform of the correlation coefficent - the spectra. From the autocorrelation function, $\rho(\tau)$, one frequently wishes to derive the integral time scale (T) defined by

$$T = \int_0^\infty \rho(\tau)\, d\tau \qquad (1).$$

There are however several well-known difficulties in the use of this equation. First of all, T depends on the properties of ρ at large lags, which are not usually well known. Moreover, T is sensitive to the method chosen for separation of large-scale and small-scale features. This requires that T be estimated by other usually unsatisfactory ways instead of directly by Eq. 1. In this paper, a new model for ρ (based on multi-scaling of turbulence) and a new technique to estimate the integral time scale are developed, which allow more information to be gathered directly from ρ and which also overcome some difficulties in the estimates of T.

2. Means and Fluctuations

Given any wind speed time series, it is common to decompose it into a mean, $\bar{U}$, and fluctuations about the mean, u, such that

$$U = \bar{U} + u \qquad (2).$$

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The averaging process must satisfy the Reynolds postulates

$$\bar{u} = 0\,; \quad \overline{\bar{U}u} = 0; \quad \bar{\bar{U}} = \bar{U} \qquad (3).$$

The relationships in Eq. 3 are satisfied when the time average over total time t is defined as

$$\bar{U} = \frac{1}{t}\int_0^t U(t')\, dt' \qquad (4).$$

An average over a shorter interval of time, Δt, can also be defined as

$$\bar{U}^i = \frac{1}{\Delta t}\int_{(i-1)\Delta t}^{i\Delta t} U(t')dt' \qquad (5).$$

With the definitions of Eqs. 4 and 5, the wind speed can be written as

$$U = \bar{U} + (\bar{U}^i - \bar{U}) + (U - \bar{U}^i) \qquad (6)$$

or in notation similar to Eq. 2

$$U = \bar{U} + u' + u'' \qquad (7)$$

where the u of Eq. 2 is now equal to u'+u". Continuing with the multi-scaling idea, an average over a sub-sub-interval can be defined as

$$\bar{U}^{ij} = \frac{1}{\Delta(\Delta t)} \int_{(i-1)\Delta t + (j-1)\Delta(\Delta t)}^{(i-1)\Delta t + j\Delta(\Delta t)} U(t')\, dt' \qquad (8)$$

and, as with Eq. 6, U can be written as

$$U = \bar{U} + (\bar{U}^{j} - \bar{U}) + (\bar{U}^{ij} - \bar{U}^{j}) + (U - \bar{U}^{ij}) \qquad (9)$$

or

$$U = \bar{U} + u' + u'' + u''' \qquad (10)$$

where now the u of Eq. 2 is equal to u'+u"+u'''. The averaging of Eqs. 4, 5, and 8 is illustrated in Fig. 1. This procedure can be continued such that

$$U = \bar{U} + u' + u'' + u''' + \cdots\cdots \qquad (11).$$

While for one set of observations, the averages defined by Eqs. 5 and 8 will be discontinuous across sub-intervals, ensemble averaging should yield a continuum.

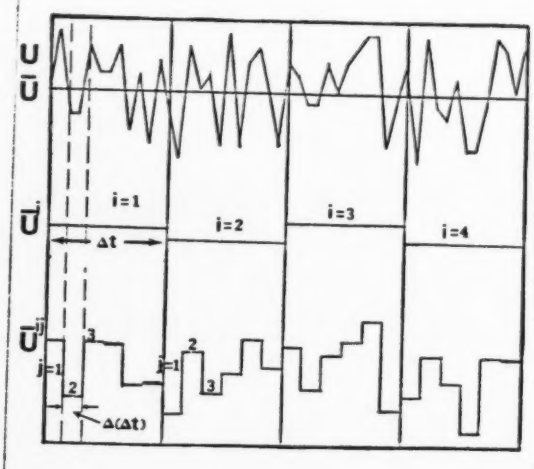

Fig. 1. Illustration of the definition of $\bar{U}$, $\bar{U}^{i}$ and $\bar{U}^{ij}$

It can be shown that averages defined in this manner satisfy the Reynolds postulates, and that the covariances of the primed quantities of different order are zero.

3. Autocorrelation Structure

The autocovariance is defined as

$$R(\tau) = \overline{u(t)\, u(t+\tau)} \qquad (12)$$

where

$$R(0) = \overline{u^2} = \sigma_u^2 \qquad (13).$$

Using Eq. 11, the autocovariance now takes the form

$$R(\tau) = \overline{\left[u'(t) + u''(t) + \cdots\right]\left[u'(t+\tau) + u''(t+\tau) + \cdots\right]} \qquad (14).$$

Since the covariances of different order (i.e. u'u", u'u''', u"u''') are zero, Eq. 14 can be shown to be

$$R(\tau) = \overline{u'(t)\, u'(t+\tau)} + \overline{u''(t)\, u''(t+\tau)} + \cdots\cdots \qquad (15)$$

or

$$R(\tau) = R_1(\tau) + R_2(\tau) + \cdots\cdots \qquad (16).$$

Note that we can also depict the variance of each scale, and that each variance is proportional to the energy of that scale of motion. From Eq. 14 when $\tau = 0$ we see that

$$\sigma_u^2 = \sigma_{u'}^2 + \sigma_{u''}^2 + \cdots\cdots \qquad (17).$$

The autocorrelation coefficient can now be defined as

$$\rho(\tau) = \frac{1}{\sigma_u^2}\left[R_1(\tau) + R_2(\tau) + \cdots\right]$$

$$= \frac{\sigma_{u'}^2}{\sigma_u^2}\rho_1 + \frac{\sigma_{u''}^2}{\sigma_u^2}\rho_2 + \cdots\cdots$$

$$= a_1\rho_1 + a_2\rho_2 + \cdots\cdots \qquad (18).$$

Since the energy spectral density (spectra) is the Fourier transform of the autocorrelation function, it can be written as

$$\Phi(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \rho(\tau)\, e^{i\omega\tau}\, d\tau$$

$$= a_1\Phi_1 + a_2\Phi_2 + \cdots\cdots \qquad (19)$$

implying that each scale of turbulence will indeed have its own spectrum with different peak frequencies. Models with two spectral density terms were proposed by Kaimal (1978) and Hojstrup (1982).

Both the autocorrelation coefficients and the spectrum of turbulence can be decomposed into a series of terms which may each reflect a different family of dominant eddy sizes. The a_i represent the percentages of the total energy in each group.

Sutton (1983) suggests that when the eddies are assumed to have the same size, the autocorrelation function has an exponential form. With multi-scaling, each group may be assumed to have eddies of approximately the same size, and thus the autocorrelation can be described by a series of exponential terms as

$$\rho(\tau) = a_1 e^{-\tau/T_1} + a_2 e^{-\tau/T_2} + \ldots . \qquad (20).$$

Substituting Eq. 20 into Eq. 19 gives the spectra as

$$\Phi(\omega) = \frac{2 a_1 T_1}{\pi(\omega^2 T_1^2+1)} + \frac{2 a_2 T_2}{\pi(\omega^2 T_2^2+1)} + \ldots \qquad (21)$$

where the T_i's represent the time scales of the eddies in that group. For example, in the boundary layer the largest vertical eddies are expected to be of size equal to the height of the mixed layer, while there may be another family whose size is of the order of the topography and others of the microscale.

From the definition of the integral time scale and Eq. 20, we see that Eq. 1 simplifies to

$$T = \sum_i a_i T_i \qquad (22).$$

Therefore, the total time scale is the weighted sum of the time scales of the individual eddy groups.

4. Dispersion Parameters

In spite of the almost universal use of tables of values for dispersion parameters whose use depends on gross synoptic observations, it has long been recognized that on site measurements of turbulence may offer improved modeling accuracy. Taylor (1921) suggested the following form for σ_y:

$$\sigma_y^2 = 2\sigma_v^2 \int_0^{\xi}\int_0^{t} \rho(\tau)\, d\tau\, dt \qquad (23).$$

when $\rho(\tau)$ is the Lagrangian autocorrelation function.

Substituting from Eq. 20 gives

$$\sigma_y^2 = 2\sigma_v^2 \sum_i a_i T_i \left[\frac{\xi}{T_i} - (1 - e^{-\xi/T_i})\right] \qquad (24)$$

where ξ is the travel time of the plume. The ratio of ξ to T_i and the weight, a_i, will determine which terms of the series are important to keep.

5. Determination of T_i

When dividing U into a series of terms, u',u'', ... etc., the choice of averaging time for each sub-interval is quite arbitrary. We can also compute the autocorrelation function from an arbitrary series of sub-intervals. The challenge is to choose them such that the sub-interval time scales T_i will have the proper physical significance. If we can select the T_i from the original data, the likelihood that they will relate to the dominant eddy sizes is much greater. We can do this by directly calculating $\rho(\tau)$ from the original measurements and decomposing it into a series of exponentials where

$$\rho(\tau) = \sum_i a_i e^{-\tau/T_i} \qquad (25).$$

To obtain results with physical significance, one would like any method that is used to fit the autocorrelation function to meet the following criteria:

i) $a_i > 0$, $T_i > 0$;

ii) $\Sigma a_i = 1$;

iii) the method selects an optimal number of terms;

iv) the method converges to a 'best' fit.

A number of techniques have been applied to the problem of fitting data with a series of exponential terms. Many of the commonly used techniques such as the Prony method (Lancos, 1956) have been applied with limited success. In this application of fitting the autocorrelation coefficient, simple methods such as that of Prony are not completely satisfactory in that the a_i and T_i terms can be negative. Wiscombe and Evans (1977) offer a much improved approach which avoids the ill conditioning of the exponential fitting problem and meets the criteria mentioned above. However, the approach has yet to be applied to the autocorrelation decomposition in the preliminary testing.

6. Example Calculation

As a preliminary test of the decomposition of an autocorrelation function, some wind data taken at 10 m in the Tennessee Plume Study (Schiermeier et al. 1979) were examined. The measurements were made during strongly convective conditions, at a sampling rate of 2 Hz. Fig. 2 shows the autocorrelation function of the vertical wind speed out to 120 lags and the fit to that function using the normal one-term exponential sum.

The two-term fit was made using the simple Prony method. A three-term fit using this method did not improve the results, and a slightly negative time scale appeared.

The best fit equation is:

$$\rho(\tau) = 0.54\, e^{-\tau/153} + 0.46\, e^{-\tau/8} \qquad (26).$$

where τ is in seconds. Since the average wind speed was 4.5 m/s, this implies length scales of 688 m and 36 m. Interestingly, an independent spectral analysis of the data also showed a peak at 150 seconds. (The smoothing of the data in the spectral analysis would not allow the 8-second peak to be seen.) For comparison, the one-term exponential fit gave a single time scale of approximately 40 seconds.

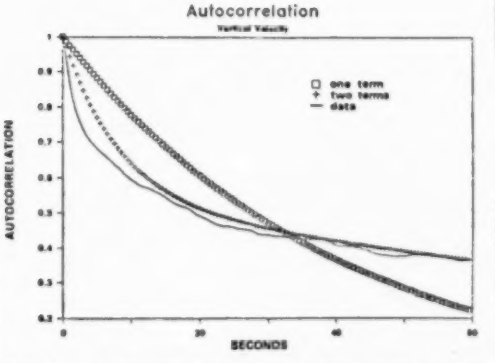

Fig. 2. Autocorrelation of 10-m vertical velocity data and approximation to the data using one and two exponential terms. The two-term fit is according to the Prony method.

7. Conclusion

Decomposition of wind data into several scales of motion shows promise as a powerful tool in the study of turbulence, and is a useful adjunct to the understanding of autocorrelation functions and spectra. It may also be very useful in the application of onsite data to diffusion models and studies of plume dispersion. There is good reason to believe that exponential fitting techniques will play a useful role in this decomposition procedure, but a great deal is left to be done before the technique is adequately demonstrated.

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